Polar coordinates

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$r = 4$ in
$\theta = \cos 2t$ in rad.

we want acceleration we $\theta = 30°$

So using polar coordinates to analyze this motion, we break motion into radial component and angular component.

the magnitude of the acceleration

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$$

r is constant, so,
$\dot{r} = 0$, $\ddot{r} = 0$.

$$\theta = \cos 2t$$
$$\dot{\theta} = -2 \sin 2t$$
$$\ddot{\theta} = -4 \cos 2t$$

for $\theta = 30° \longrightarrow \pi/6$ radians

~~$\dot{\theta}$~~ this occurs at what time?

$$\pi/6 = \cos 2t$$
$$\cos^{-1}(\pi/6) = \cos^{-1}(\cos 2t)$$
$$2t = 1.0197$$
$$t = 0.5098 \text{ s.} \approx 0.51 \text{ s.}$$

at $t = 0.51$,

$$\dot{\theta} = -2 \sin(2 \cdot 0.51) = -1.7$$

$$\ddot{\theta} = -4 \cos(2 \cdot 0.51) = -2.09$$

$$a = \sqrt{\left(0 - 4(-1.7)^2\right)^2 + \left(4(-2.09) + 0\right)^2}$$

$$a = 14.27 \text{ in/s}^2$$

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$\dot{\theta} = 0.2$ rad/s

$\ddot{r} = 0.5$ m/s^2

after $t = 3$ s:

Velocity

radial component:

$V_r = \dot{r}$

the bug started from rest at 0.5 m/s^2

at $t = 3$, $V_r = 0 + 0.5(3) = 1.5$ m/s

transverse component

$v_\theta = r\dot{\theta}$ = ~~1.5(3)(0.2) = 0.9 m/s~~

$= 2.25\,(0.2) = 0.45$ m/s

(2.25 → $ut + \frac{1}{2}at^2$ or $\frac{v^2 - u^2}{2a}$)

Acceleration

radial

$a_r = \ddot{r} - r\theta^2$

~~Erabs~~

$$r = \frac{v^2 - u^2}{2a} = \frac{1.5^2 - 0}{2(0.5)} = 2.25 \text{ m}$$

$a_r = 0.5 - 2.25\,(0.2)^2 =$ ~~0.22~~ 0.4 m/s^2

transverse components

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 1.5(0) + 2(1.5)(0.2)$

$= 0.6$ m/s^2

Cylinderical components

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$r = 0.5\ m$

$\theta = 0.5t^3$

$z = 2 - 0.2t^2$

So we need to find the radial (r), transverse (θ) and z - components of the motion

Velocity :

radial

$\dot{r} = 0$ because r is a constant value

transverse

$r\dot{\theta} = \overset{0.5}{0.5} \cdot 3t^2 =$ ~~$1.5t^2$~~ $0.75t^2$

z .

$z = 2 - 0.2t^2$

$\dot{z} = -0.4t$

when $\theta = 2\pi$

$2\pi = 0.5t^3$

$t^3 = 2\pi / 0.5$

$t = 2.32\ s.$

at this time

$\dot{r} = 0$

$r\dot{\theta} = 0.75(2.32)^2 =$ ~~8.07336~~ 4.0368

$\dot{z} = -0.4(2.32) = -0.928$

magnitude of velocity $= \sqrt{0^2 + 4.0368^2 + (-0.928)^2}$

$=$ ~~8.02267 m/s~~ $= 4.142\ m/s$

acceleration

~~[illegible]~~ .

~~[illegible]~~

radial component is given by

$\ddot{r} - r\dot{\theta}^2 = 0 - 0.5 . (0.5 . 3t^2)^2 =$ ~~[illegible]~~ $-1.125t^4$

transverse component

$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5 . 1.5 . 2t + 0 = 1.5t$

z component

$\ddot{z} = -0.4$

magnitude of acceleration at $\theta = 2\pi$ which corresponds to $t = 2.32$ s.

~~[illegible]~~

$a = \sqrt{\left[-1.125(2.32)^4\right]^2 + \left[1.5(2.32)\right]^2 + (0.4)^2}$

$= 32.99 \ m/s^2$