Cartesian Vectors
(aka 3-d Vectors)

2 – 60

We have two forces acting in our 3-dimensional reference frame.

Find projection of the forces onto the three axes. We shall later sum up components to obtain the resultant.

Force 1.
Note that given the set up we will have to project onto the x-y plane, an then project to the relevant axis to get the component along that axis.

$$F_{1x} = -450 \cos 45 . \sin 30 = -159.01\,N$$

$$F_{1y} = 450 \cos 45 . \cos 30 = 275.57\,N$$

$$F_{1z} = 450 \sin 45 = 318.2\,N$$

Force 2.
Given the set up we have angle α to the x-axis, β to y-axis. What we need is angle to z-axis, γ.
We can get this from the identity

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 45 + \cos^2 60 + \cos^2\gamma = 1$$

$$\Rightarrow \gamma = 60°$$

$F_{2x} = 600 \cos 45 = 424 \text{ N}$

$F_{2y} = 600 \cos 60 = 300 \text{ N}$

$F_{2z} = -600 \cos 60 = -300 \text{ N}$

Resultant force F_R

$F_{Rx} = F_{1x} + F_{2x} = 265.16 \text{ N}$

$F_{Ry} = 575.57 \text{ N}$

$F_{Rz} = 18.2 \text{ N}$

Magnitude of F_R

$$|F_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$$

$$= 633.97 \text{ N}$$

Direction angles

$$\alpha = \cos^{-1}\left(\frac{F_{Rx}}{|F_R|}\right) = \cancel{265.16}\ \cos^{-1}\left(\frac{265.16}{633.97}\right) = 65.$$

likewise for

β

γ

Cartesian Vectors

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$$F_{1x} = -60 \cos 50 . \cos 30 = -33.4 \text{ lb}$$

$$F_{1y} = 60 \cos 50 \sin 30 = 19.28 \text{ lb}$$

$$F_{1z} = -60 \sin 50 = \cancel{\text{[illegible]}} \; -45.96$$

direction angles

$$\alpha = \cos^{-1}\left(\frac{F_{1x}}{|F_1|}\right) = \cos^{-1}\left(\frac{-33.4}{60}\right) = \cancel{\text{[illegible]}} \; 123.83^\circ$$

$$\beta = \cos^{-1}\left(\frac{F_{1y}}{|F_1|}\right) = \cos^{-1}\left(\frac{19.28}{60}\right) = \cancel{\text{[illegible]}} \; 71.26^\circ$$

$$\gamma = \cos^{-1}\left(\frac{F_{1z}}{|F_1|}\right) = \cos^{-1}\left(\frac{-45.96}{60}\right) = \cancel{\text{[illegible]}} \; 139.99^\circ$$

Cartesian Vectors

2-73

$F_{2x} = 300 \cos 60 = 150\ N$

~~$F_{2y} = 300$~~

$F_{2z} = 300 \cos 60 = 150\ N$

now

$\cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1$

$\Rightarrow \cos^2 \beta_2 = 1 - (\cos^2 \alpha_2 + \cos^2 \gamma_2)$

$= 1 - (2 \cos^2 60)$

$= 0.5$

$\Rightarrow \beta_2 = 45°$

$F_{2y} = 300 \cos 45 = 212.1\ N$

$F_{3y} = -200\left(\frac{4}{5}\right) = 160\ N$

$F_{3z} = 200\left(\frac{3}{5}\right) = 120\ N$

Resultant:

$R_x = 150 + 400 = 550\ N$

$R_y = 212.1 + (-160) = 52.1\ N$

$R_z = 150 + 120 = 270\ N$

$|R| = \sqrt{550^2 + 52.1^2 + 270^2} = 614.91\ N$

$\alpha = \cos^{-1}\left(\frac{550}{614.91}\right) = 26.5°$