Equilibrium of a Rigid Body

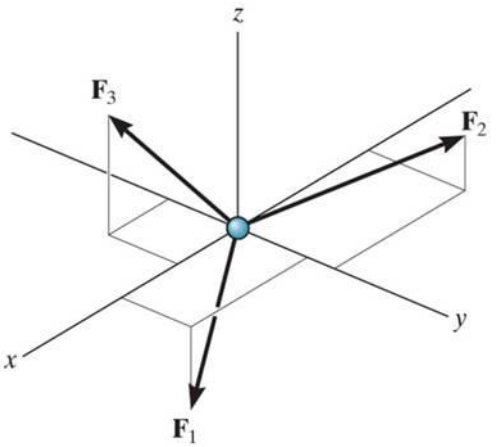
Chapter 5

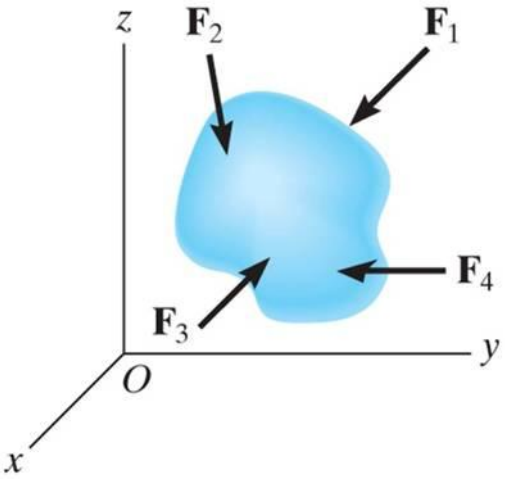
Overview

- Rigid Body Equilibrium
- Free Body Diagrams
- Equations of Equilibrium
- 2 and 3-Force Members
- Statical Determinacy

CONDITIONS FOR RIGID-BODY EQUILIBRIUM

- Recall forces acting on a particle
- In contrast forces on a rigid body acting on a rigid body may not be concurrent, and cause moments, which will cause rotation
- For a rigid body to be in equilibrium, sum of forces and sum of moments about some point will equal.





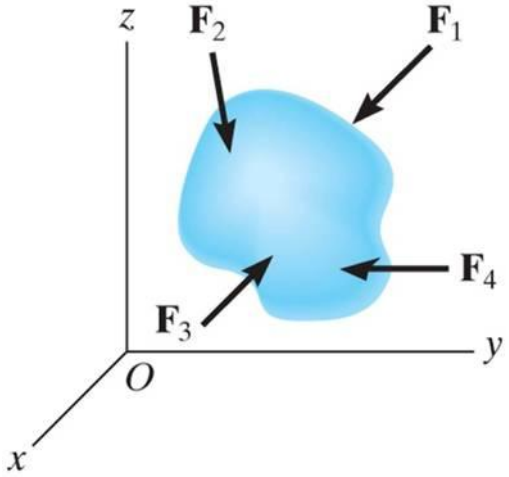
CONDITIONS FOR RIGID-BODY EQUILIBRIUM

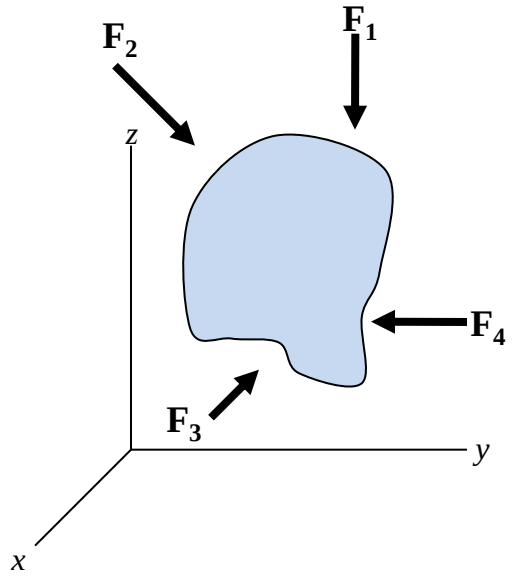
In other terms, for body at rest

- $\sum \boldsymbol{F} = 0$ (no translation)

And

- $\sum \boldsymbol{M}_O = 0$ (no rotation)
- Otherwise?

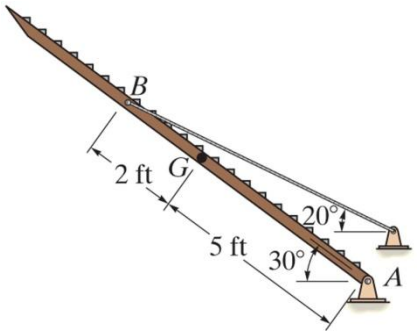


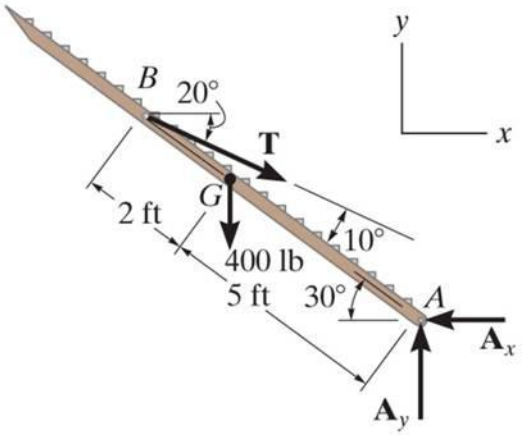


SOLVING RIGID BODY EQUILIBRIUM PROBLEMS

Using the ramp system shown we may go through the following steps

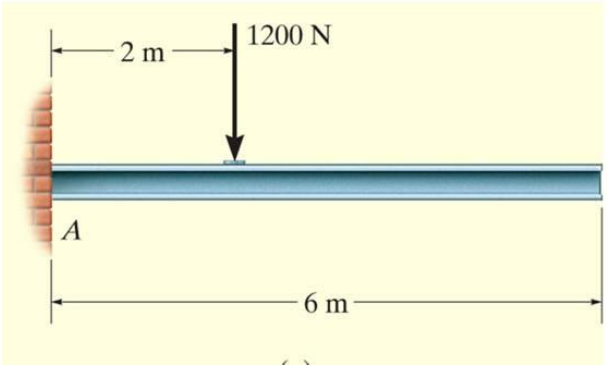
1. Create an idealized model
2. Draw the free body diagram (show all active and reactive forces
3. Apply the equations of equilibrium to solve for the unknowns

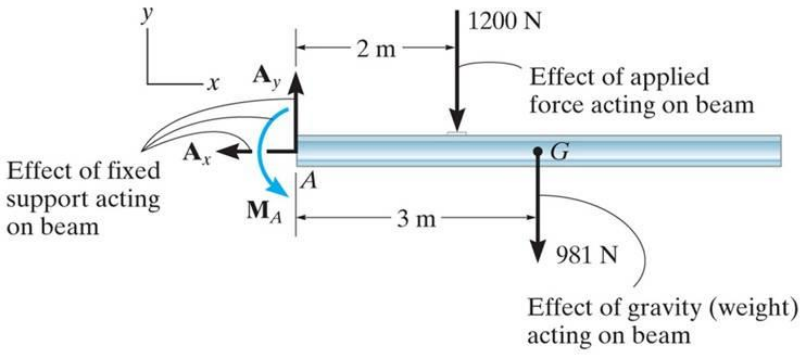




SOLVING RIGID BODY EQUILIBRIUM PROBLEMS

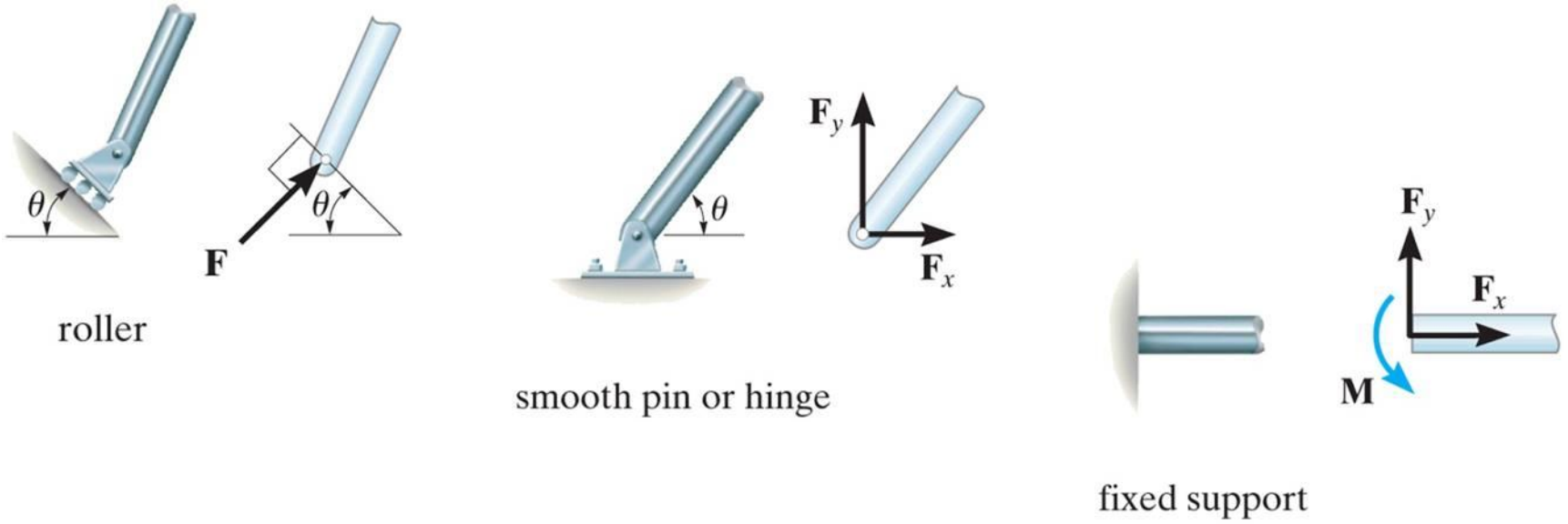
- Idealized model: Imagine the body to be isolated or cut "free" from its constraints and draw its outlined shape.
- External forces and couple moments: typically include: applied loads, support reactions, weight of the body.
- Label all forces and moments with their magnitude and show direction of application





2-D Support Reactions

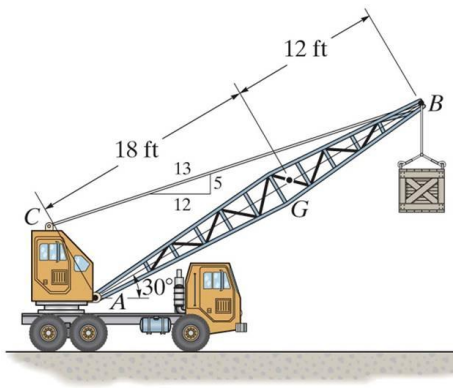
- How is the rigid body fixed in place and what are the reactive forces holding it up?
- There are several types of support reactions in use. Some examples are shown
- Table 5-1 in text gives exhaustive illustration

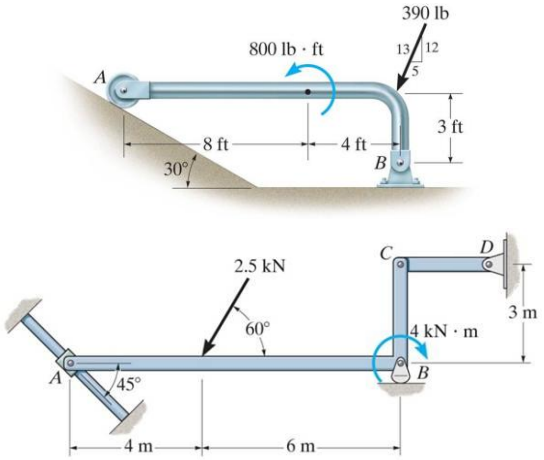


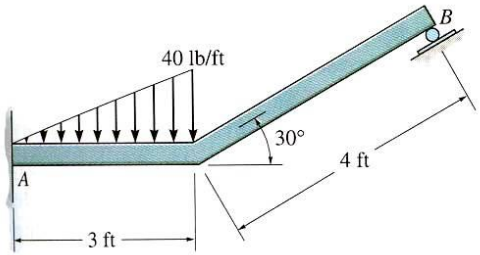
2-D Support Reactions

General rules of thumb:

- if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction.
- if rotation is prevented, a couple moment is exerted on the body in the opposite direction.







Questions and Comments ?

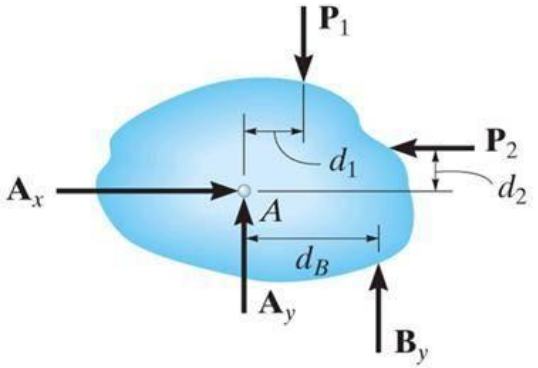
Two and Three – Force Members

- Consider a rigid body subject a system of forces the x-y plane.
- This 2-D equilibrium condition can be represented by the three scalar equations:

- $\sum F_x = 0$

  $\sum F_y = 0$

  $\sum M_O = 0$

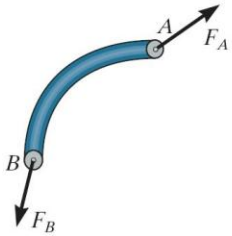


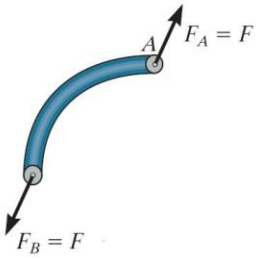
where point O is any arbitrary point.

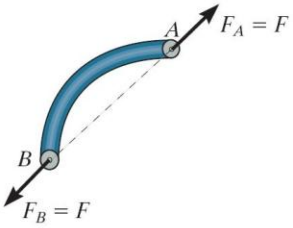
Two and Three – Force Members

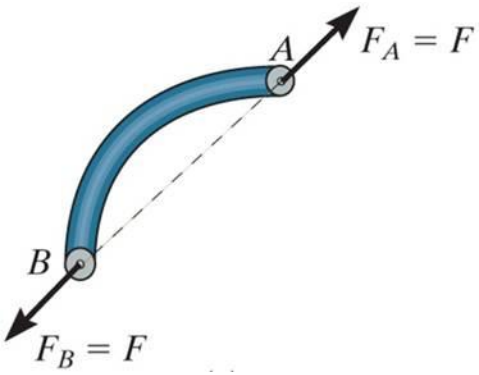
- The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B).
- If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.

Two and Three – Force Members









Two-Force Members

Some problem solving tips

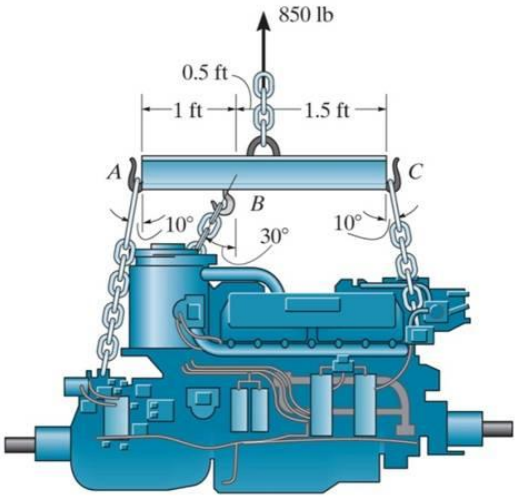
- If not given, establish a suitable x - y coordinate system.
- Draw a free body diagram (FBD) of the object
- Apply the three equations of equilibrium to solve for the unknowns.

Two-Force Members

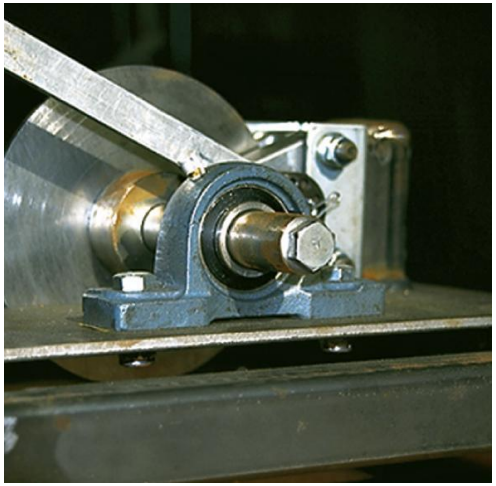
Some Notes:

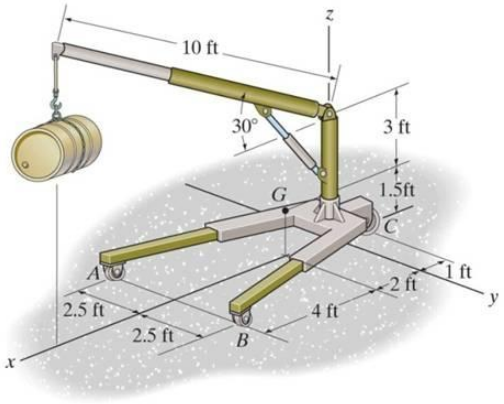
- If there are more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics
- The order in which we apply equations may affect the simplicity of the solution.
- If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.

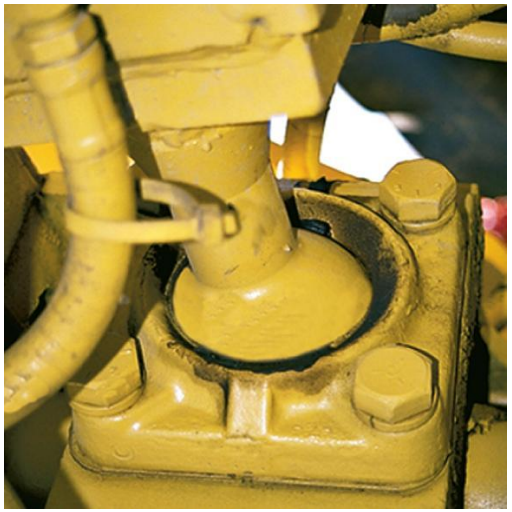
Questions and Comments ?



3-D Free Body Diagrams





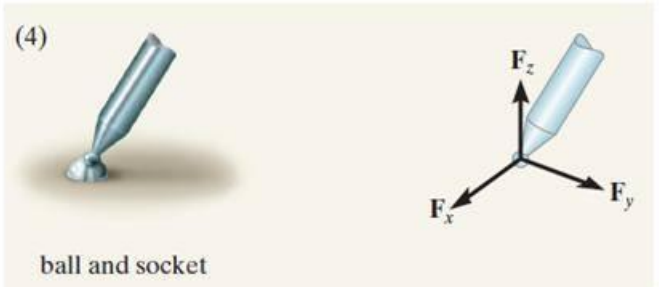


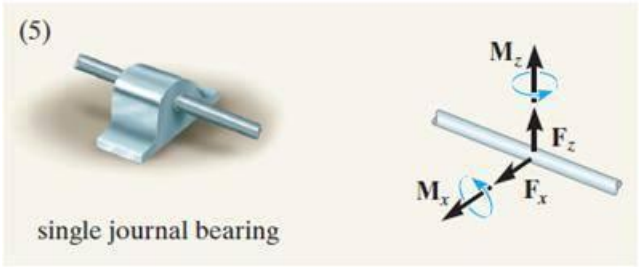
3-D Support Reactions

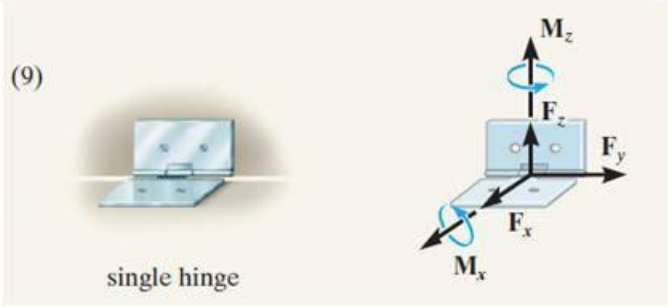
- if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body.
- if rotation is prevented, a couple moment is exerted on the body by the support.

3-D Support Reactions

- Some types of 3-D supports (See table 5-2 in text for exhaustive list)
- if rotation is prevented, a couple moment is exerted on the body by the support.

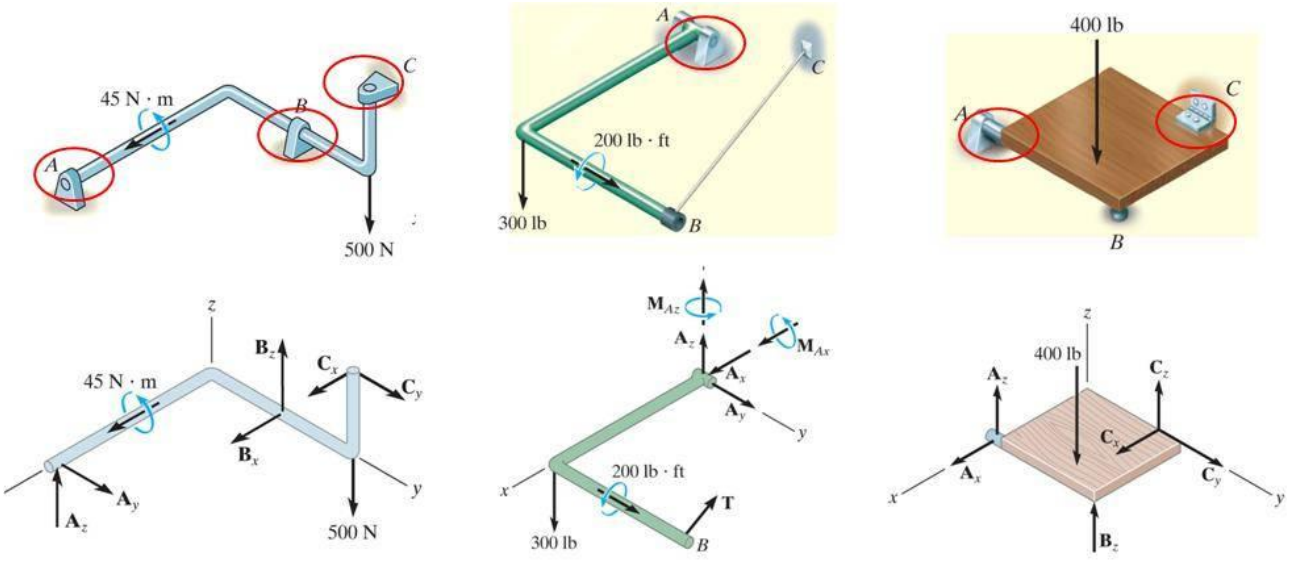






3-D Supports

- A single bearing or hinge can prevent rotation by providing a resistive couple moment.
- it is usually preferred to use two or more properly aligned bearings or hinges.
- in these cases, only force reactions are generated and there are no moment reactions created.



3-D Equations of Equilibrium

- $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$
- Therefore

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

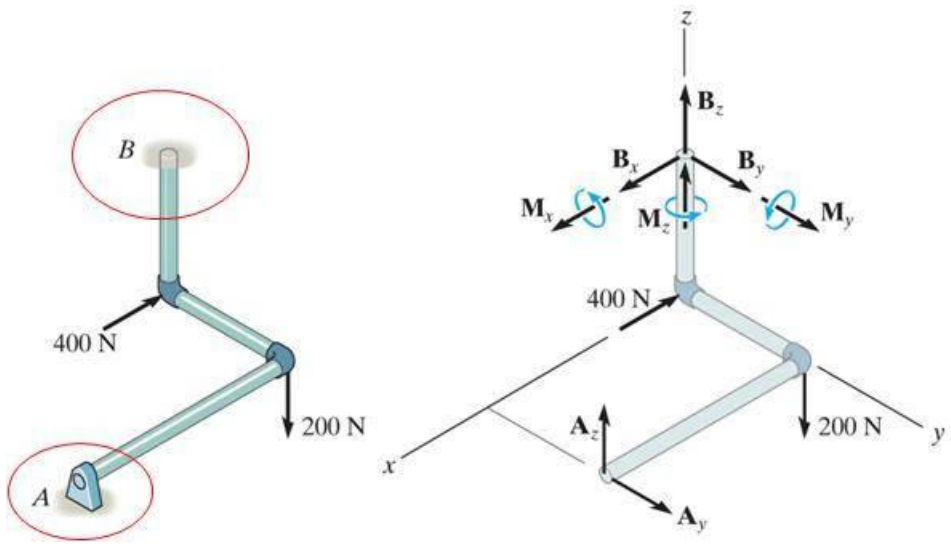
$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

- The moment equations can be determined about any point.
- choosing the point where the maximum number of unknown forces are present simplifies the solution.

3-D Equations of Equilibrium

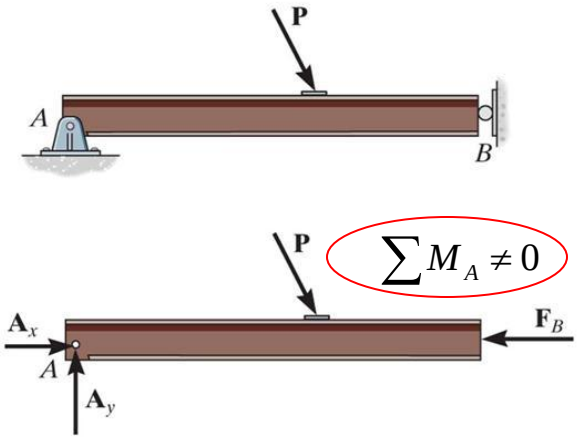
- Any forces occurring at the point where moments are taken do not appear in the moment equation since they pass through the point.
- Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.
- A problem that is statically indeterminate has more unknowns than equations of equilibrium.

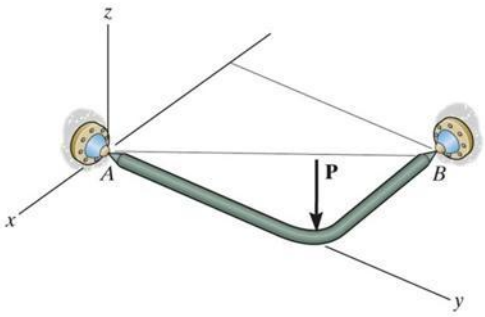
Statically Indeterminate Systems

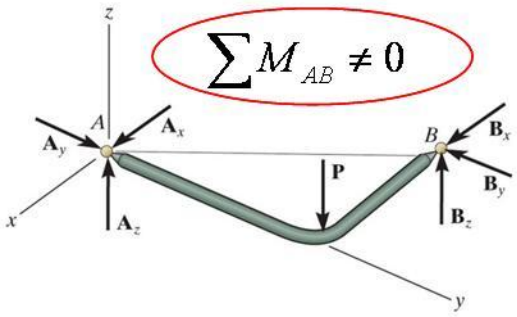


Improper Constraints

- In some cases, there may be as many unknown reactions as there are equations of equilibrium.
- However, if the supports are not properly constrained, the body may become unstable for some loading cases







Questions and Comments ?