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claim: average bulb lifetime is smaller than 750 hours

$H_0: \mu = 750$

$H_a: \mu < 750$

$\alpha = 0.05$

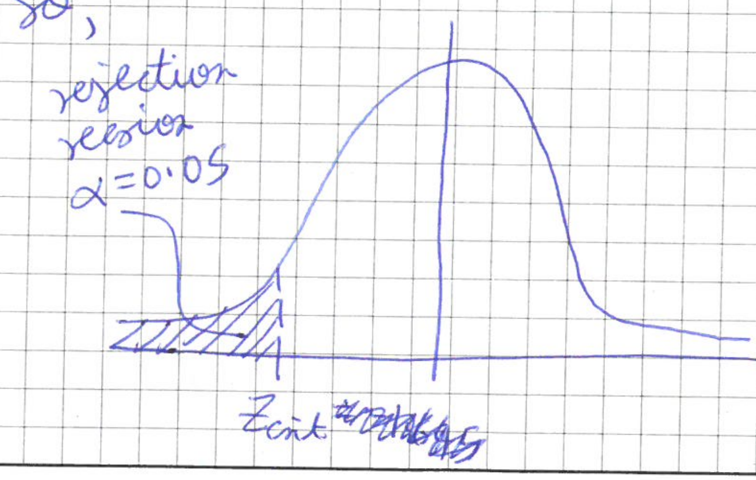
From the ~~data~~ sample data we have

$n = 50 \longrightarrow$ z-table

$\bar{X} = 738.44$ hours

$S = 38.2$ hours

so,



for $\alpha = 0.05$ to the left

we go to table and find the

Z-score

$$\alpha = 0.05 \longrightarrow Z = -1.645$$

So $Z_{crit} = -1.645$

Test statistic

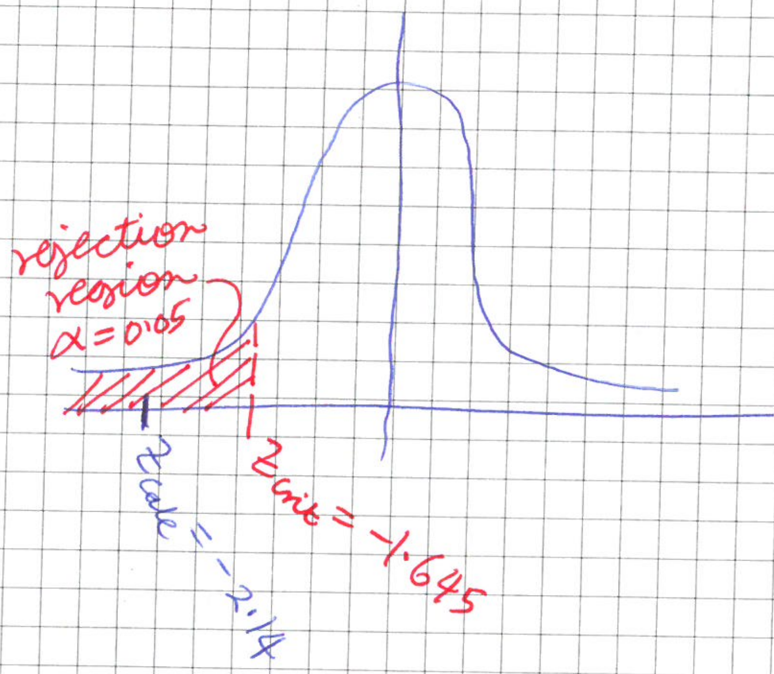
$$Z_{calc} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

$$= \frac{738.44 - 750}{38.2/\sqrt{50}}$$

$$= -2.14$$

so lets plot Z_{calc} in relation to

Z_{crit} on the bell curve.



So our Z_{calc} falls in rejection region. So we REJECT H_0!

So according to this data, and at 95% confidence the true average bulb life time will be less than 750 hours. We conclude that ~~the~~ our claim is true.

Therefore the claim in the advertisement is FALSE!

Alternate Method

The p-Value Method

$H_0 : \mu = 750$

$H_a : \mu < 750$

$\alpha = 0.05$

$n = 50 \longrightarrow z - \text{table}$

$\bar{x} = 738.44 \;, \; s = 38.2$

Test statistic

$$Z_{calc} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{738.44 - 750}{38.2/\sqrt{50}}$$

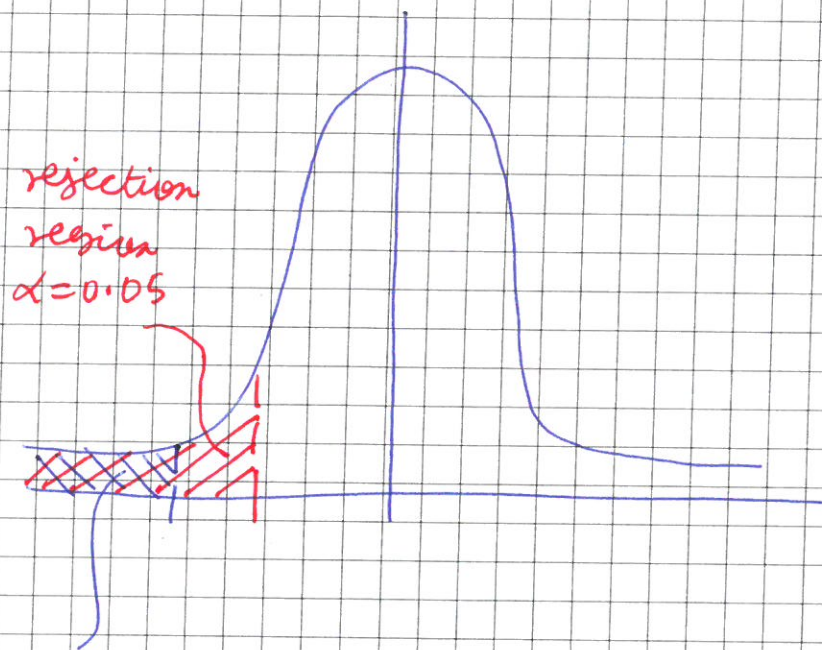
$$= -2.14$$

Now we go to the table and find the α-value associated with this Z-value (to the left of it because we have a "$<$" condition.

$$Z_{calc} = -2.14 \longrightarrow \alpha_{calc} = 0.0162$$

we called α_{calc} the observed α-value (as in observed in the data). It is also called the p-value.

We now plot the p-value in relation to the α-significance value. Note that α, p-values are areas shaded under the curve



α_{calc} or p-value $= 0.0162$

So our p-value is 'stuck' in the rejection region.

So we reject H_0!

We conclude that the true population is less than 750 hours.

So the claim in the advertisement is not true.

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claim ; the true population mean ~~is~~ differs from 100

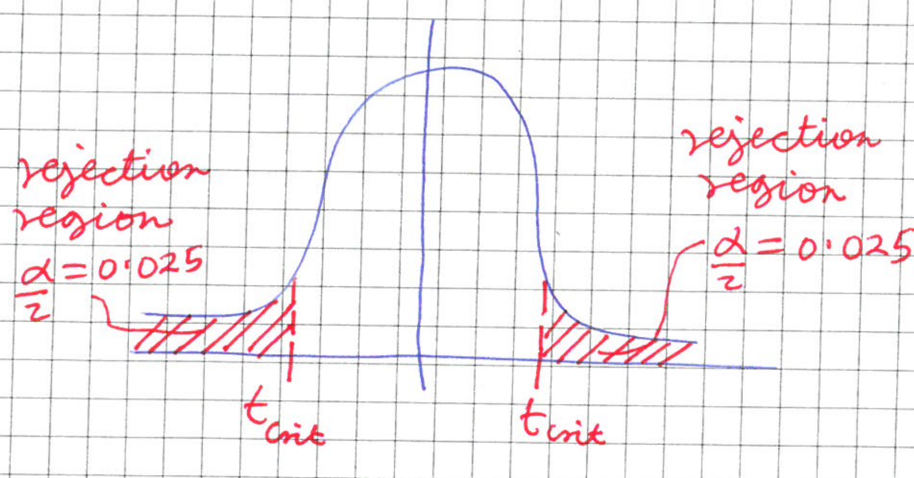
$H_0 : \mu = 100$

$H_a : \mu \neq 100$ (2-tail test)

$\alpha = 0.05$

$n = 12, \quad \bar{x} = 98.375, \quad s = 6.11$

Small sample so t-table



$t_{crit} = t_{\frac{\alpha}{2} = 0.025,\ \nu = 11\ df}$

Solns

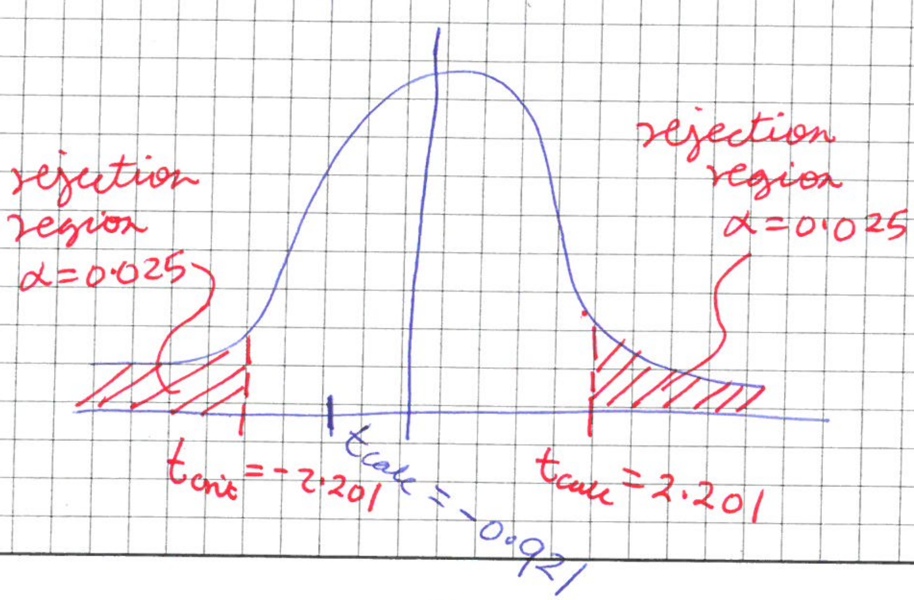
1. Critical Value Method

so $t_{crit} = t_{0.025,\,11\,df} = 2.201$

$$t_{calc} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{98.375 - 100}{6.11/\sqrt{12}}$$

$$= -0.921$$



So our t_{calc} is not in the rejection region

So we FAIL TO REJECT H_0!

So μ is indeed 100. The claim that μ differs from 100 is false.

P – Values Method

So from

$$t_{calc} = -0.921$$

go to t-table and find associated α-value (which we call p-value).

at 11 df.

t	0.921	1.363	1.796
	↓	↓	↓
α	?	0.10	0.05

$$\frac{\alpha - 0.10}{0.921 - 1.363} = \frac{0.1 - 0.05}{1.363 - 1.796}$$

$$\Rightarrow \quad \alpha_{calc} = 0.151$$

or

$$p\text{-value} = 0.151$$

We shade from the left because

out t_{calc} was a negative.

p-value is not enclosed in

rejection region. It 'spills' out

of rejection region.

So we fail to reject H_0!

So the claim is not valid.

$\mu = 100$ units

Confidence Interval

This method of testing a hypothesis

is relevant only for 2-tail tests

$\alpha = 0.05$

So we need to construct

a 95% CI

If our null hypothesis value (100) falls in the confidence interval, we fail to reject H_0, otherwise we reject H_0.

$$95\%\ CI = \bar{X} \pm t_{\frac{\alpha}{2}, \nu = n-1} \cdot \frac{S}{\sqrt{n}}$$

$$= 98.375 \pm t_{0.025, 11} \cdot \frac{6.11}{\sqrt{12}}$$

$\rightarrow$ from table $= 2.201$

$$= 98.375 \pm 2.201 \frac{(6.11)}{\sqrt{12}}$$

$$= 98.375 \pm 3.882$$

$$= [94.493, 102.257]$$

$\mu_0 = 100 \in [94.493, 102.257]$

So we FAIL TO REJECT H_0

The claim is false.